5.9. Translation Variations: Universals

1. Single-Predicate Universal Sentences. English variations on "all" include "**every**," "**each**," and "**any**".

> Everything in the universe is material .
> **All** objects in the universe are material
> **Each** object in the universe is material.

> Every object can be destroyed.
> **Any** object can be destroyed.

But here caution is required; for within the context of a larger negation, "any" instead expresses an *existential* claim.[1]

> Nothing is a unicorn.
> There are**n't any** unicorns.
> There is**n't any** object which is a unicorn.

Concerning combinations of quantifiers and negations, the English order is often a good guide to which should come first in formal translation (and so have wider scope).[2]

<u>Not</u> <u>even one object</u> is destructible

$$\mathbf{\sim\exists x\ Gx}$$

<u>Some object</u> is <u>in</u>destructible

$$\mathbf{\exists x \sim Gx}$$

<u>Not</u> <u>all objects</u> are destructible

$$\mathbf{\sim\forall x\ Gx}$$

<u>All objects</u> are <u>in</u>destructible

$$\mathbf{\forall x \sim Gx}$$

[1] As noted in *5.7. Translation Variations: Existential Sentences*
[2] As noted in *5.5. A First Look at Quantifier Semantics*

2. Two-Predicate Universal Sentences. We face new complexity when moving from quantified sentences with a single predicate – "material," or "travels faster than light" – to those with several. Formal translation will, for example, count **two predicates** in the following sentence.

All tigers are striped.

We can recast this two-predicate universal sentence in 'technical English' by appealing to the dummy term "object" as the quantificational hook on which to hang two predicates.

"All **tigers** are **striped**."
All objects which **are tigers** are **striped** objects.

The puzzle here is the proper way of formally linking the quasi-sentences "x is a tiger" and "x is striped".

Our first guess might be to conjoin them together (and then universally quantify that conjunction), just as we did with two-predicate existential sentences.

☠ Proper Translation? ☠

All objects which **are tigers** are objects which **are striped**.
(For every object in the universe, the following holds of it:
it is a tiger, ***and*** *it is striped*.)

For all x: *x is a tiger*, ***and*** *x is striped*

G: is a tiger **H**: is striped

$\forall x\,(Gx \wedge Hx)$

But "$\forall x\,(Gx \wedge Hx)$" makes a much stronger claim than the English original. For in saying of each thing that it's striped and a tiger, we end up saying that **everything is a striped tiger**.

Note that intuitively, in a situation where there are just two tigers – say, Hobbes and Daniel – and both those tigers are striped, the sentence "All tigers are striped" should be true. But it's easy to build a model like that where the universal sentence "∀x (Gx ∧ Hx)" is still false.

G__: is a tiger **H__**: is striped

D: {**Hobbes**, **Daniel**, **Jack**}

A: **Hobbes**	**G**: {**Hobbes**, **Daniel**}
B: **Daniel**	**H**: {**Hobbes**, **Daniel**}
C: **Jack**	

In this model "∀x (Gx ∧ Hx)" has three instances – one of which is **false**.

1 (GA ∧ HA)
1 (GB ∧ HB)
0 (GC ∧ HC)

We wanted to assert the stripedness of all tigers; but here we assert the tigerhood and stripedness of all objects.

Instead we need first to restrict the domain of discussion to the tigers, and then say of just those that they're all striped. That is: we say of each object that it's striped **assuming it's a tiger**. For that reason a **conditional** is the proper way of stringing together the two little quasi-sentences; and adding a universal quantifier yields the formal translation.

The Proper Translation

"**All tigers are striped**."

(For every object in the universe, the following holds of it:
assuming *it's a tiger*, *it's striped*.)

For all x: if *x is a tiger*, *x is striped*

G: is a tiger **H**: is striped

$\forall x\,(Gx \rightarrow Hx)$

Note that the semantics for universal (and conditional) sentences supports this translation. Returning to the earlier model, the formal sentence "$\forall x\,(Gx \rightarrow Hx)$" will have three instance, each a conditional.

G__: is a tiger **H__**: is striped

D: {**Hobbes**, **Daniel**, **Jack**}

A: **Hobbes**	G: {**Hobbes**, **Daniel**}
B: **Daniel**	H: {**Hobbes**, **Daniel**}
C: **Jack**	

Instances of "$\forall x\,(Gx \rightarrow Hx)$" in this model:

1 $(GA \rightarrow HA)$
1 $(GB \rightarrow HB)$
0 $(GC \rightarrow HC)$

The two tigers in this model are both striped, making the first two instances true (because when both antecedent and consequent are true, the whole conditional is true). And Jack acts here the way any non-tiger will toward such a conditional: making the antecedent false (since it's false that Jack's a tiger), the whole conditional comes out (trivially) true.

On this approach to translation, the only sort of object that would make the sentence "All tigers are striped" false would be a tiger which isn't striped. And that seems like the right result.

As a further variation on "all" in multi-predicate cases, we can use "**whatever**". So the following two sentences are equivalent.

> **All objects** <u>having mass</u> <u>exert gravitational attraction</u>.
> **Whatever** <u>has mass</u> <u>exerts gravitational attraction</u>.

English also employs **tacit** or **implicit** (unspoken but understood) **quantification**, where a universal sentence is intended even though no quantifier phrase appears. So the following two sentences make the same claim, though the first leaves off the universal quantifier.

> Tigers are striped.
> All tigers are striped.

As a rule of thumb for recognizing such tacit quantification in English, we can attach the words "in general" or "as a rule" before a sentence, and ask if it means the same as the original. The following sentences, for example, say basically the same thing; so we conclude that there is a tacit universal quantifier in the first sentence.

> Tigers are striped.
> In general: tigers are striped.

But adding such a quantifying phrase to the next sentence yields a result meaning something quite different from the original; so we conclude that the first sentence does not use tacit quantification.

> Tigers are in Sector Four.
> In general: tigers are in Sector Four.
> As a rule: tigers are in Sector Four.

(The sentence “Tigers are in Sector Four” is instead making an existential claim: it means the same as “There are some tigers in Sector Four”.)

Just as with existential sentences, we can restrict the discussion to tigers with the phrase “**among**”.

Among tigers, all are striped.

G: is a tiger **H**: is striped

$\forall x\,(Gx \rightarrow Hx)$

This yields a new way of translating the sentence “Some but not all doctors are men” and “Only some doctors are men”.

G: is a doctor **H**: is a man

$(\exists x\,(Gx \wedge Hx) \wedge {\sim}\forall x\,(Gx \rightarrow Hx))$

Note that “**only**” acts as a universal quantifier phrase of a peculiar sort. We remarked in Chapter Four that adding “only” to a conditional phrase serves to **switch the order of parts** in a conditional: whereas “Q” is the antecedent in “P if Q,” “P” is the antecedent in “P only if Q”.

The same holds with quantification: “All G are H” employs a conditional with “G” as antecedent and “H” as consequent. But for “Only G are H” we instead use a conditional with “H” in the antecedent and “G” in consequent.

G: is a human **H**: is a language user

All humans are language users.

$\forall x\,(Gx \rightarrow Hx)$

Only humans are language users.

$\forall x\,(Hx \rightarrow Gx)$

If only humans are language users, then anything using language will be human – that is, "All language users are human".

Recalling that a conditional and its converse often make quite different claims, we see why "All language users are human" and "All humans are language users" likewise make non-equivalent claims. For example: there may be no non-human language users, but also even some humans aren't language users. Such a scenario makes "All language users are human" true, but "All humans are language users" false.

We can combine comparable "all" and "only" claims into a 'universalized' biconditional.

"All and only humans are language users."

G: is a human **H**: is a language user

$(\forall x\,(Gx \rightarrow Hx) \;\land\; \forall x\,(Gx \rightarrow Hx)\,)$

$\forall x\,(Gx \leftrightarrow Hx)$

3. Universals and Negation. In the discussion of existential sentences we translated a negative existential sentence, such as "No sparrows are carnivores," as the negation of an existential sentence.

I: is a sparrow **J**: is a carnivore

No sparrows are crnivores.
$\sim\exists x\,(Ix \wedge Jx)$

But universal quantifiers offer us a second way of translating negative existentials. For wherever it's true that "No sparrows are carnivores" it's true that "All sparrows are non-carnivores" (or "All sparrows fail to be carnivores").

$\forall x\,(Ix \rightarrow \sim Jx)$

Tacit quantification can be used here as well; so the following sentences are likewise translated identically.

No sparrows are carnivores.
Sparrows aren't carnivores.

Treating negative existentials according to this translation variation explains a familiar semantic phenomenon in a new way. Recall further that **a conditional is equivalent to its contrapositive** – for example, "$(P \rightarrow Q)$" is equivalent to "$(\sim Q \rightarrow \sim P)$". But then "$\forall x\,(Ix \rightarrow \sim Jx)$" will be equivalent to the following "$\forall x\,(\sim\sim Jx \rightarrow \sim Ix)$"**.** Clearing double negations from the antecedent shows that "$\forall x\,(Ix \rightarrow \sim Jx)$" ("No sparrows are carnivores") is equivalent to "$\forall x\,(Jx \rightarrow \sim Ix)$" ("No carnivores are sparrows"). Once again: **order of parts does not affect the truth** of a two-predicate negative existential.

And recognizing the equivalence of the sentences “$(P \rightarrow Q)$” and “$\sim(P \wedge \sim Q)$,” we can (using Quantifier Negation and Double Negation) explain the equivalence of our two ways for translating negative existentials.

“$\forall x\,(Ix \rightarrow \sim Jx)$” is logically equivalence to “$\sim\exists x\,(Ix \wedge Jx)$”
“$\forall x\,(Jx \rightarrow \sim Ix)$” is logically equivalence to “$\sim\exists x\,(Jx \wedge Ix)$”

Indeed, every sentence expressible in the formal language using existential quantifiers finds an equivalent translation using universal quantifiers instead. The traditional (two-predicate) **Square of Opposition** illustrates these equivalences.

Square of Opposition

(A) All G are H
$\forall x\,(Gx \rightarrow Hx)$
$\sim\exists x\,(Gx \wedge \sim Hx)$

(E) No G are H
$\forall x\,(Gx \rightarrow \sim Hx)$
$\sim\exists x\,(Gx \wedge Hx)$

(I) Some G are H
$\sim\forall x\,(Gx \rightarrow \sim Hx)$
$\exists x\,(Gx \wedge Hx)$

(O) Some G are not H
$\sim\forall x\,(Gx \rightarrow Hx)$
$\exists x\,(Gx \wedge \sim Hx)$

Universal Translation Variations
Summary Sheet

All objects are G
Everything is G
Each thing is G
Anything is G
} $\forall x\ Gx$

Nothing is G
There are no Gs
There aren't any Gs
} $\forall x \sim Gx$

All G are H
Everything G is H
Each G is H
Whatever is G is H
Among G, all are H
(In general,) G are H
} $\forall x\ (Gx \rightarrow Hx)$

Some but not all G are H
Only some G are H
} $(\exists x\ (Gx \land Hx) \land \sim\forall x\ (Gx \rightarrow Hx))$

Only G are H } $\forall x\ (Hx \rightarrow Gx)$

All and only G are H } $\forall x\ (Gx \leftrightarrow Hx)$

No G are H
There are no GH
All G are non-H
} $\forall x\ (Gx \rightarrow \sim Hx)$